Physics Qualifying Exam September 29, 1986

THERMAL PHYSICS AND CLASSICAL MECHANICS

Instructions:

(1) You may bring either Goldstein's Classical Mechanics text or Landau and Lifshitz's Classical Mechanics text. No notes or other books are allowed.

(2) Problems 1 through 5 are thermal physics problems. Please do problems 1 through 3 and either problem 4 or problem 5. Do not do both. Credit will not be given for both so indicate clearly on the cover of your Blue Book which one you want graded.

(3) Problems 6 through 8 are classical mechanics problems. Do all three.

(4) Solve problems 1 through 3 and 4 or 5 in one Blue Book. Start a second Blue Book for problems 6 through 8. DO NOT PUT SOLUTIONS TO THERMAL PHYSICS PROBLEMS AND CLASSICAL MECHANICS PROBLEMS IN THE SAME BLUE BOOK.

(5) The thermal physics section, Problems 1 - 3 and 4 or 5 are worth a total of 50 points. The classical mechanics section, problems 6 - 8 is also worth 50 points. The exam is three hours long and you may divide your time between the two sections as you see fit.

(6) The point values for the individual problems appear in paranthesis next to each problem.

(7) Only 1 grade will be given for the entire exam.

Example: Consider Students A, B and C. If A receives 50 points on thermal physics and 0 points on classical mechanics, and B receives 0 points for thermal physics and 50 for classical mechanics, and C receives 25 points for each part, their performances on the written exam will be considered identical. However, the oral examination committee may notice the differences in performance and plan their questions accordingly.

(2) e) At about what temperature, and for what reason, do you expect your answer to c) (and b) for that matter) to break down if the gas were cooled?

Now the partition is pulled upward allowing the gas to fill both halves of the box. No gas escapes.

(1) f) Carefully explain what happens to the internal energy of the gas and why.

(1) g) What happens to the temperature of the gas?

(1) h) Calculate the change in entropy of the gas.

(1) i) Discuss the relationship between h) and the well-known expression $đQ = TdS$.

(8) (Total) 2. Make numerical estimates of

(4) a) the average distance between the molecules in the air filling this room.

(4) b) the mean free path between intermolecular collisions of the air in this room.

(12) (Total) 3. A rock with a heat capacity of 300 J/K and a mass of 1 kg is put into a lake whose temperature is 20°C.

(4) a) Calculate the <u>total</u> increase in entropy, after thermal equilibrium is established, (of the rock plus the lake) if the rock is originally at 20°C and it is dropped into the lake from the top of a 500 m cliff.

(5) b) Calculate $C_A(2A_o, T)$.

(5) c) Calculate $C_\sigma(A_o, T)$.

(5) d) Calculate the Helmholtz free energy in terms of its natural variables.

(For convenience let $\sigma(A = A_o, T = 0) = 0$ and $F(A = A_o, T = 0) = 0$.)

IF YOU DID PROBLEM 4, DO NOT DO PROBLEM 5.

(20) (Total) 5. A thin wire is stretched between two rigid clamps a distance L apart. The velocity of a wave on the wire is C, independent of wavelength. We wish to calculate the contribution of the standing wave normal modes of the wire to its thermal properties.

(4) a) Calculate the partition function Z, of a single mode.

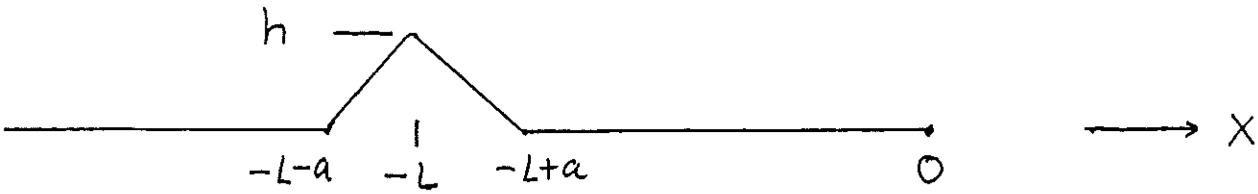
(4) b) Calculate the total partition function of all the modes.

(4) c) Calculate the free energy of the modes, Evaluate any product or sum that appears by turning it into an integral over a dimensionless variable. Call the value of that integral I and write your answer to c) and d) in terms of I. You need not evaluate I by doing the integral.

(4) d) Use your answer to c) to calculate the heat capacity at constant length of the modes on the stretched wire.

(4) e) What happens to the heat capacity of the modes if the tension in the wire is doubled?

d) Consider the following initial displacement (t = 0) with zero initial velocity, centered about x = -L with a << L, for the case of the string fixed at x = 0.



Draw the shape of the string at time $t = \frac{1}{2}\frac{L}{C}$.

Draw the shape of the string at time $t = \frac{3}{2}\frac{L}{C}$.

(20) 8. Find the orbit (r as a function of ϕ) of a particle of mass m bound in a spherical harmonic oscillator potential. Use the method analogous to the Coulomb orbit solution. (No credit will be given for the x-y method.) Give the reason behind all of your assumptions. Begin by finding the minimum energy possible for a given angular momentum.

The following integral will be of use for the quadrature.

Integrals Involving $X^{1/2} = (ax^2 + bx + c)^{1/2}$

380.001.
$$\int \frac{dx}{X^{1/2}} = \frac{1}{a^{1/2}} \log |2(aX)^{1/2} + 2ax + b|, \qquad [a > 0],$$
$$= \frac{1}{a^{1/2}} \sinh^{-1} \frac{2ax + b}{(4ac - b^2)^{1/2}}, \qquad \begin{bmatrix} a > 0, \\ 4ac > b^2 \end{bmatrix},$$
$$= \frac{1}{a^{1/2}} \log |2ax + b|, \quad [b^2 = 4ac,\ a > 0,\ 2ax+b > 0]$$
$$= \frac{-1}{a^{1/2}} \log |2ax + b|, \quad [b^2 = 4ac,\ a > 0,\ 2ax+b < 0],$$
$$= \frac{-1}{(-a)^{1/2}} \sin^{-1} \frac{(2ax + b)}{(b^2 - 4ac)^{1/2}}, \qquad \begin{bmatrix} a < 0, \quad b^2 > 4ac, \\ |2ax + b| < (b^2 - 4ac)^{1/2} \end{bmatrix}.$$

The principal values of $\sin^{-1}$, between $-\pi/2$ and $\pi/2$, are to be taken.

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MATHEMATICAL PHYSICS

Do all six problems. All problems carry equal weight.

One textbook and a set of integral tables are allowed in the examination room.

1. Evaluate

$$S = \sum_{n=-\infty}^{\infty} \frac{1}{(2n+1)^2+a^2}$$

by contour integration.

2. Maradudin polynomials of the second kind, $M_n(x)$, $n = 0,1,2,\ldots,$ are defined by the generating function

$$\frac{x - t}{(1-xt)^2} = \sum_{n=0}^{\infty} t^n M_n(x) \quad .$$

What is the three term recurrence formula these functions satisfy?

3. Estimate the smallest value of λ for which the equation

$$\frac{d^2y}{dx^2} - \frac{3}{x}\frac{dy}{dx} + \lambda y = 0 \qquad 0 < x < 1$$

$$y(0) = 0, \quad y(1) = 0$$

has a solution. (Hint: Rewrite this equation in the form of a Sturm-Liouville equation and use a variational method. For this it is helpful to know, à la Frobenuis, how $y(x)$ goes to zero as $x \to 0$.)

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QUANTUM MECHANICS

3 Hours

You are allowed one quantum mechanics book, one book of mathematical tables and a calculator; no notes.

Do Problem 1, and three of the remaining five problems. All problems carry equal weight.

We will be available to answer questions 10 minutes after you have received the problems, and then periodically -- at about 40 minute intervals.

Problem (1): Compulsory

(a) In the Born approximation, determine the differential cross section for scattering from a potential of the form

$$V(r) = V_o \exp(-r/a).$$

(b) Sketch the angular variation of the differential cross section in the two limits $k\,a \ll 1$, $k\,a \gg 1$, where k is the wave vector of the particle incident on the potential.

(c) Again in the Born approximation, find an expression for the total cross section for scattering from the potential given above.

Do three out of the following.

Problem (4):

Treat the $\ell = 0$ bound state of a proton and neutron (deuteron) by describing their interaction by means of the attractive potential $V(r) = -V_o e^{-\frac{r}{a}}$. Use a Schrödinger equation with the reduced mass $m^* = \frac{1}{2} m_N$, ($m_N \cong 938$ MeV), and for the "radius" a take 2.18 fm.

(a) By using an exact solution of the Schrödinger equation, determine the constant V_o, in MeV, so as to match the experimental binding energy of the deuteron, $E_D = -2.23$ MeV. (Hint: let $u = e^{-\frac{r}{a}}$).

(b) Use a Rayleigh-Ritz variational calculation with a one parameter family of exponential trial functions, $\phi_\alpha(r) = N^{-1}\exp(-\alpha r/2a)$, to determine the binding energy approximately, using the value $V_o = 32.7$ MeV which emerges from part (a), and compare with the experimental value.

Useful mathematical info: $J_1(\mu) = 0$ for $\mu = 3.83$ (first zero of Bessel function)

Useful numbers (use units of MeV, fm, s):

$$\hbar = 6.6 \times 10^{-22} \text{ MeV}\cdot\text{s}$$

$$m_N \cong m_p \cong 938 \text{ MeV.}$$

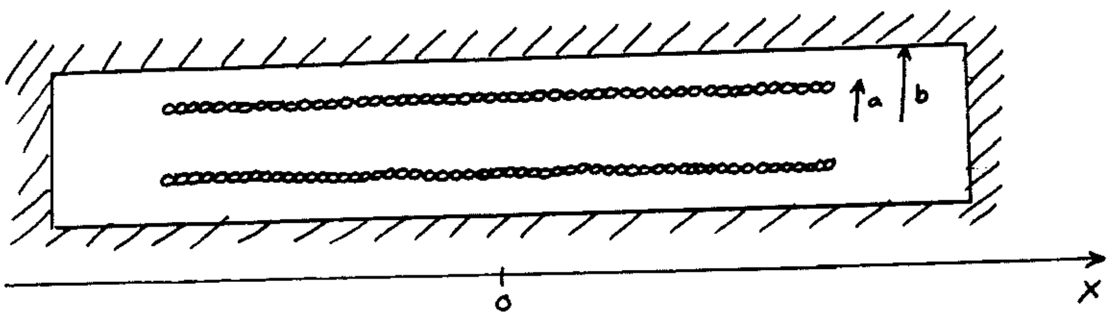
Physics Qualifying Examination

30 September 1986
8:30 - 11:30 a.m.

ELECTRICITY AND MAGNETISM

The examination is three hours in duration and is to be taken without the use of notes. However, you may use Jackson's "Classical Electrodynamics." Do five of the six problems. Each problem is worth a maximum of 20 points. In the event that you cannot complete a problem, outline in as much detail as possible how you would go about solving the problem if more time were available.

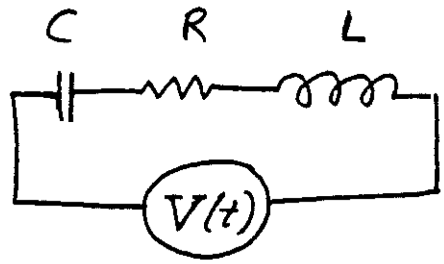
1.



A current I flows in a long uniform air core solenoid of length ℓ, radius a, and total number of turns N. The solenoid is centered in a long cylindrical cavity within a superconductor, as shown above. The cavity has radius b and length ℓ', (For our purposes a superconductor is to be considered simply as a material which perfectly excludes magnetic fields, but note that surface currents will be induced in the superconductor.)

Assume: $\ell' \gg \ell \gg a$ and $\ell \gg b$ (i.e., neglect end effects).

3.



The above RLC circuit is underdamped $\left(R < 2\sqrt{\frac{L}{C}}\right)$. It is driven by a time-varying external voltage given by:

$$
\begin{aligned}
V(t) &= 0 \qquad -\infty < t < 0 \\
&= V_o \qquad 0 < t < T \\
&= 0 \qquad T < t < \infty
\end{aligned}
$$

where T is unrelated to the RLC parameters.

Find a Green's function such that the charge on the capacitor C at time t is given by

$$q(t) = \int_{-\infty}^{\infty} G(t,t')V(t')dt' .$$

Use this to find explicit expressions for the current I flowing in the circuit at an arbitrary time $t > T$.

a) Derive an expression for the reflectivity of the slab.

b) What is the minimum non-zero thickness that the slab can have in order for the reflectivity to vanish? Take $n = 1.5$.

6. A rigid, straight wire of length R rotates about an axis perpendicular to it and passing through one end. The plane in which the wire moves is normal to a uniform magnetic field of magnitude B. Contact is made to the wire at the axle and at the free end by means of a rail in the form of a circular arc. The angular speed of rotation of the wire is ω. What is the induced EMF in the wire?